**Problem Set 5**

*Due Friday, October 11, 2013 before 9:31am*

*Instructions. This is a GROUP assignment. Please form yourselves into groups of no more than THREE students.*

*You might find Question 6 to be time-consuming. I suggest that you start this question before Tuesday night.*

*Our excellent Teaching Assistant Sicheng GUO suggests that you submit your computations in the form of an Excel file(s) or Python Notebook via Canvas. Because you can enter text into these files, add labels to the file, format it for readability, etc., I think it makes sense put your entire solution into the Excel or Python file, and submit just a single file. I also add the following suggestions:*

*(a) The names of the group members should be clearly marked in your submissions*

*(b) Please make sure that your file is well-organized and clear, with appropriate text, explanations, and formatting.* ***If Mr. Guo cannot figure out what you did, then what you did is wrong.***

*Remark. This assignment is an excellent opportunity for you to teach yourself how to use the Python’s graphing functions. It is also an excellent opportunity to familiarize yourself with some of Python’s other capabilities.*

1. (mean-variance optimization) Data on canvas provide the expected annual returns, standard deviations of annual returns, and correlations of returns for the leading stock indexes in 7 different countries. You are a U.S.-based investor, and you are able to invest in the portfolios underlying these stock indexes. Interpret the returns as the U.S. dollar returns (not the local currency returns), so that the tables show the properties of returns that are available to you.
2. Please find the mean-variance frontier (actually, the mean-standard deviation frontier) when there is no risk-free asset, for values of E[*rp*] between 0% and 20%. In doing this, assume that you may short-sell any of the indexes. Prepare a graph showing the mean-variance (standard deviation) frontier for E[*rp*] between 0% and 20%.

*Remark*. Even though I call it the mean-variance frontier, the horizontal axis should be the standard deviation.

1. What is the standard deviation of the minimum variance portfolio? What is the vector of portfolio weights of the minimum variance portfolio?
2. Next, impose the constraints that: (i) you may not short-sell any of the indexes, i.e. *w*i ≥ 0 for *i* = 1, …, 7. Please find the mean-variance frontier when there is no risk-free asset, for values of E[*rp*] between 0% and 20%. (For many values of E[*rp*] there might be no portfolio that both satisfies the constraints on *wi* and achieves the value of E[*rp*] .) Prepare a graph showing the mean-variance frontier for E[*rp*] between 0% and 20%. **By considering the weights in part a) explain why your frontier in part c) is so different from part b).**
3. Next, assume that you may also invest in a risk-free asset that has an interest rate of 0.5%/year, and that you may short-sell any of the indexes. Please find the mean-variance frontier, for values of E[*rp*] between 1% and 20%. Prepare a graph showing the mean-variance frontier for E[*rp*] between 1% and 20%. What is the Sharpe ratio?
4. Any point on the mean-variance frontier in part (d) can be attained by combining the risk-free asset with a single portfolio of risky assets (remember the “One Fund Theorem.”) What are the portfolio weights in the risky asset portfolio? (Remember that the portfolio weights of the risky asset portfolio should sum to 1.) What is the expected return of the risky asset portfolio? If you want to attain an expected return of 20%, what should be the weights in the risk-free asset and the risky asset portfolio?
5. In class, we considered an example in which the covariance matrix of the returns on the 11 stocks was estimated using only 12 monthly returns, from January 2023 – December 2023. Now, estimate the covariance matrix using **11** monthly returns, February 2023– December 2023. (The data are in the spreadsheet used to construct the examples we discussed in class.) The estimated covariance matrix will be singular, which implies that it is possible to find a portfolio of the 11 stocks that (according to the estimated covariance matrix) has no risk. Please find such a portfolio of the 11 stocks that has no risk. What is the return on this portfolio? Please graph the mean-variance frontier, for values of E[*rp*] between m0 and 1.67% (as in class), where m0 is the return on the riskless portfolio of the 11 stocks.
6. The tab “Returns of various assets” in the Excel spreadsheet “Q3\_data” includes the monthly returns on the S&P 500 index, the NASDAQ index, an index of NYSE-listed small capitalization stocks, an index of NYSE-listed large capitalization stocks, and two hedge fund indexes, the HFRI Equity Market Neutral index and the HFRI High Yield index.
7. Using all of the available data (December 1990-May 2005), compute the average monthly returns and standard deviations of monthly returns for these 6 indexes. Plot the six points on a graph that shows standard deviation on the horizontal axis and mean return on the vertical axis.
8. Using these 6 asset classes and assuming that there is no risk-free asset, find and graph the mean-variance (standard deviation) frontier. What is the combination of mean and standard deviation corresponding to Fairfield Sentry? Where is it relative to the mean-variance frontier?